THE LOCAL FRICTION IN THE INITIAL PART OF A PLANE-PARALLEL CHANNEL

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Results are presented on the local friction in the initial part of a plane-parallel channel with laminar flow and also for the stabilization zone.

There are many theoretical and experimental studies on the flow in the initial sections of tubes; many heat exchangers and similar systems employ short tubes, for which one cannot employ relationships based on the assumption of fully stabilized conditions.

In 1891, Bussineskii considered the length of the initial section in a pipe that was required to establish a parabolic velocity distribution; his solution implied that the length of the stabilization region was equal to the length within which the tangential stress at the wall reached a steady value.

Various viewpoints have been taken on the initial section in a plane parallel tube in papers [1-3]. The experimental results are restricted to information derived by pressure measurements on the initial parts [4, 5]. Direct measurements of the local friction at the start have not been published, and here we describe some such measurements.

The experiments were done by an electrochemical method, which consisted in measuring the limiting current in an electrochemical cell formed by two electrodes, the power supply, and an electrolyte. Levich [6] has given the theory of a diffusion process at a polarized electrode, and this has been repeatedly tested and widely used in simulating heat and mass transfer processes [7]; in 1966, this method was used [8] to measure tangential stress.

The cathode consisted of a piece of platinum foil 100μm thick and 1-2 mm wide, which was fitted into the wall of the channel with the long side perpendicular to the flow direction. The anode lay in the exit chamber and was a platinum wire with a surface area about 100 times that of the cathode. The solution was 2 M NaOH containing 0.01 M of $K_3Fe(CN)_6$ and 0.01 M of $K_4Fe(CN)_6$. The equation for the diffusion boundary layer is solved subject to the usual boundary conditions and certain simplified assumptions [9] to give a direct relationship between the tangential stress at the wall and the saturation current:

$$\tau_0 = \frac{1.87\mu L I^3}{A^3 \Phi^3 C_+^3 D^2} . \tag{1}$$

This formula has been given before [8, 9] and has been used here to determine the friction via the measured current; the tests on the local friction were done with the apparatus shown in Fig. 1.

The main element was the flat channel (Fig. 2) 1600 mm long and of rectangular cross-section with a side ratio of 90 mm/10 mm = 9; this channel was demountable and consisted of two plates forming the top and bottom and two inserts that controlled the height and served as the sides. All parts of the channel were made of lurite. The upper plate had holes to take the electrochemical transducers, the first being 200 mm from the inlet and the others at intervals of 200 mm from this.

The electrolyte entered the channel from an input chamber, whose main purpose was to produce a uniform flow at the input; it consisted of a smoothly convergent shape with a 20-fold compression of the flow in height. Pulsations in the chamber were suppressed by honeycomb grids and textile filter layers.

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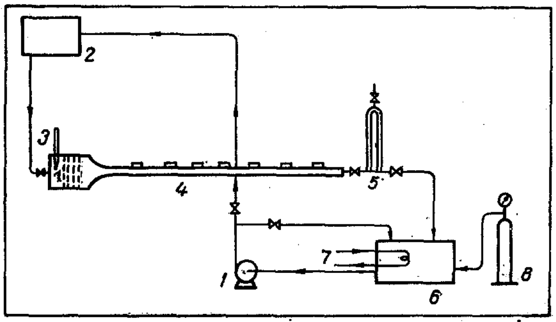

Fig. 1. Apparatus: 1) centrifugal pump; 2) constant-head tank; 3) thermometer; 4) working section of channel with friction meters; 5) flow measuring plate with U-shaped manometer; 6) overflow tank; 7) heat exchanger connected to thermostat; 8) nitrogen cylinder.

The temperature in the inlet chamber was recorded by a standard thermometer, while the solution was treated with a flow of nitrogen to remove the dissolved oxygen.

Figure 3 shows the theoretical circuit of the electrical measuring system.

The tests were done in regions of Reynolds number from 1650 to 4800, where Re was reckoned from the mean flow-rate velocity and the doubled height of the channel. Figure 4 gives the distribution of the local friction at the inlet; it also gives the distribution of the tangential stress along the channel as calculated from the following formula [1]:

$$\tau_0 = -\mu \frac{4U^2}{3h(U-U_0)}, \tag{1}$$

and from the following formula [2]:

$$\tau_0 = -\mu \frac{2U_0}{h}\left[3+2\sum_{k=1}^{\infty} \exp\left(-\frac{8\gamma_k^2 x}{\mathrm{Re}\, h}\right)\right]. \tag{2}$$

It follows from (2) and (3) that the friction in the initial section greatly exceeds the steady-state values, and also that the tangential-stress stabilization length increases considerably with Re. Formulas (2) and (3) involve the assumption of equivalence for the length of the initial part needed for friction stabilization and establishment of a parabolic velocity profile.

Most studies have been concerned with the initial part only from the viewpoint of a parabolic velocity profile throughout the section, although in many applied problems of heat and mass transfer it is more important to know the distribution of the friction in the initial section. Also, such evidence is of considerable independent importance for the hydrodynamics of tubes. The only paper [10], which is mentioned in [11], to consider this from the frictional viewpoint deals with turbulent flow conditions; it was also pointed out [11] that one should distinguish the initial length for production of a parabolic profile from that for stabilization of the tangential stress, it being considered that the second should be less than the first, though no detailed evidence on this was available.

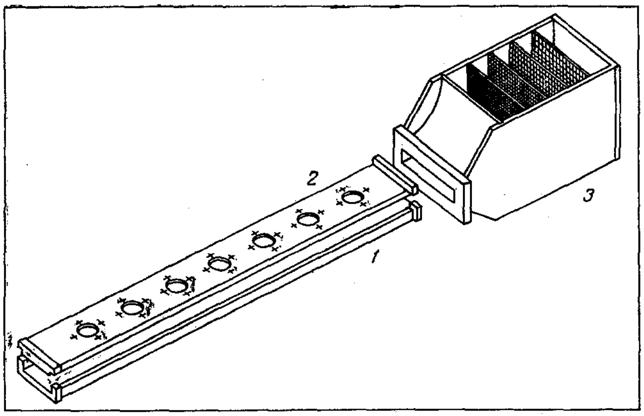

Fig. 2. Basic unit of channels: 1) lower plate; 2) upper plate with holes for probes; 3) inlet chamber with honeycombs.

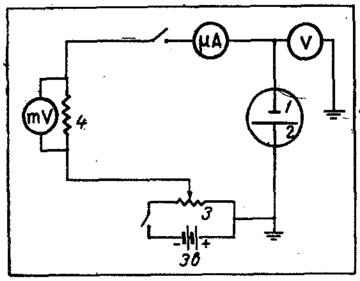

Fig. 3

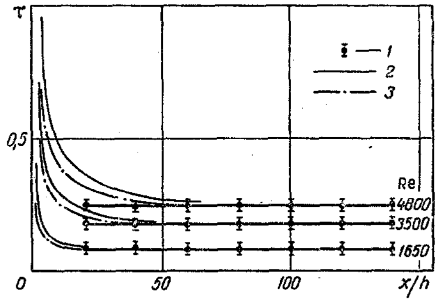

Fig. 4

Fig. 3. Measuring system: 1) electrochemical friction meter (cathode); 2) anode; 3) voltage divider of 100 ohm; 4) standard resistance of 1000 ohm.

Fig. 4. Friction distribution along channel wall: 1) experimental, 2) calculation from (3) of [2], 3) calculation from (2) of [1].

Our experiments supplemented the evidence on the friction in the initial part of such a channel for laminar flow; the error in measuring τ_0 via (1) in this range of Re was estimated as $\pm 10\%$. The velocity profile at the inlet is sufficiently rectilinear for one to be able to compare the results rigorously with existing theories of the initial part plane-parallel channels, since in all such treatments the flow is considered to develop from a rectilinear velocity profile at the inlet. Also, an essentially rectilinear profile was obtained from an evaluation of the initial velocity distribution under considerably worse conditions corresponding to the problem solved by Schlichting for laminar flow in a narrowing channel. It is clear from Fig. 4 that by a distance of 20 h from the inlet, the friction has become steady and attained the value corresponding to laminar flow, so a distance of 20 h may be taken as the limit for friction stabilization in all laminar conditions. For comparison we note that the calculation of τ_0 from (2) gave the friction as stabilizing in a distance of 65 h for Re = 4800.

The preliminary calibration of the transducers to a distance of 120 h from the input had been performed under essentially steady-state conditions, as was clear from the experimental results, which do not conflict with existing theories.

In our tests on the initial part, we did not observe tangential stresses greatly exceeding the values corresponding to completely stabilized conditions and as predicted by existing theories; the experimental results indicate the need to revise theories of the initial part as regards friction stabilization in plane-parallel channels.

NOTATION

τ_0 is the shear stress averaged over the area of electrochemical probe;
q is the steady-state mass flow in the measuring system;
D is the molecular diffusivity of ferricyanide ions;
μ is the dynamic viscosity;
L is the electrode size along flow direction;
I is the diffusion current;
A is the probe area;
Φ is the Faraday number;
C_+ is the concentration of ferricyanide ions in flow core;
U_0 is the mean flow rate velocity;
U is the local velocity flow core;
γ_k is the roots of equation $\tan\gamma = \gamma$;
h is the height of channel;
x is the distance from intersection;
Re is the Reynolds number.

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